NIS-ALT

NASA

7N-90-TM

277411

P.32

INSTITUTE FOR SPACE STUDIES

TABLES OF PLANE GALACTIC ORBITS WITH SPIRAL FIELD

K. Grossman

C. Yuan

(NASA-TM-103028) TABLES OF PLANE GALACTIC ORBITS WITH SPIRAL FIELD (NASA) 32 p

N90-70758

Unclas

00/90 0277411

GODDARD SPACE FLIGHT CENTER

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

TABLES OF PLANE GALACTIC ORBITS

WITH SPIRAL FIELD

K. Grossman
Institute for Space Studies
Goddard Space Flight Center, NASA
New York, N. Y.

and

C. Yuan
City College of New York
New York, N. Y.

TABLES OF PLANE GALACTIC ORBITS WITH SPIRAL FIELDS

Preface

This set of tables forms a sequel to the set "Tables of Plane Galactic Orbits" by Contopoulos and Strömgren, published by the Institute for Space Studies, NASA, in 1965. The 1965 tables were prepared for the purpose of quickly locating the place of origin of a star now in the solar vicinity when its age and its present-day velocity components are known. It was realized even then that the omission of the spiral gravitational field could cause significant deviations in the computated place of origin, but it was decided to have these tables prepared as a first approximation to the desired result, since no reliable information about the spiral field was available at that time. The spiral structure of the galaxy became better understood after Lin and Shu (1964, 1966) developed Bertil Lindblad's density wave concept into a definite mathematical theory; but it was only around the time of the Noordwijk Symposium, after Lin's extensive stay at the Institute for Advanced Study, Princeton (1965-66), that we realized that the pattern speed of the spiral density waves in the Galaxy should be in the neighborhood of 11-14 km/sec-kpc, rather than any value close to the angular speed of the circular motion in the solar vicinity. Still, it was difficult to decide upon a precise value for the pattern speed, the precise form of the spiral wave, its orientation (phase angle), and the intensity of the spiral gravitational field. Furthermore, there may be the need to include a group of such waves.

By using the kinematical data on stars then available and through patient experimentation with different sets of numerical values by Yuan, a single two-armed spiral pattern, with uniform pitch angle $i = 6\overset{\circ}{.}2$ and travelling at an angular speed $\Omega_p = 13.5$ km/sec-kpc, was finally adopted in the paper by Lin, Shu, and Yuan (1969). Such a spiral field (with other parameters specified in the paper cited) is superposed on the symmetrical field used by Contopoulos and Strömgren (1965) for the study of the migration of stars, and successful results were obtained.

The compatibility between the angular speed $\Omega_p = 13.5$ km/sec-kpc and the pitch angle $i = 6\overset{\circ}{.}2$ was a matter of some concern. It was resolved only after Shu developed a theory to account for the effect of the finite thickness of the galactic disk. This theory was based on certain plausible approximations, but its validity was confirmed by Vandervoort (1969) on a more rigorous basis. Thus, it was only after the Basel Symposium that complete confidence could be given to the compatibility of our method of analysis.

In view of the uncertainties in the determination of the parameters associated with the spiral pattern, we have decided to provide tables for a range of these parameters. We have **not** included in these tables the case of zero spiral field ($\eta = 0$), since our results agreed with the original tables by Contopoulos and Strömgren. As more data on individual stars became available, there may be a need to re-evaluate the determination adopted by Lin, Yuan, and Shu. In the meantime, those values appear to be satisfactory for the analysis of existing data.

The tables may also be used to construct the frequency distribution of local stars of various age groups, in the velocity space, after a spiral pattern is adopted. This has been carried out by Yuan. The results can be used for direct comparison with kinematical data.

No effort has been made to include the effect of a density distribution in the Orion arm. Analytical studies of such a correction should perhaps be made to supplement the information given in the following tables.

The patient work of Dr. Yuan and Dr. Grossman in experimentation and in compiling these tables requires extremely high skill, and the task was excellently completed. We hope that these tables will prove valuable to people interested in the migration of local stars and their places of origin.

The work is carried out at the Institute for Space Studies, NASA, New York City and at the Massachusetts Institute of Technology, where the work of C. C. Lin and C. Yuan is supported in part by grants from the National Science Foundation and the National Aeronautics and Space Administration. We wish also to thank Dr. Robert Jastrow for his support of this program.

B. Strömgren
University of Copenhagen, Denmark, and
Institute for Space Studies, NASA, NYC.

C. C. Lin
Massachusetts Institute of Technology
Cambridge, Mass.

November, 1969

<u>References</u>

Contopoulos, G. and B. Strömgren (1965)
Tables of Plane Galactic Orbits
Institute for Space Studies, NASA, N. Y.C.

Lin, C. C. and F. H. Shu (1964)
On the Spiral structure of disk galaxies
Astrophysical Journal, <u>140</u>, pp. 646-655

Lin, C. C. and F. H. Shu (1966)
On the spiral structure of disk galaxies, II
Outline of a theory of density waves
Proceedings of the National Academy of Sciences, <u>55</u>, pp. 229-234

Lin, C. C., C. Yuan and F. H. Shu (1969)
On the spiral structure of disk galaxies, III
Comparison with observations
Astrophysical Journal, <u>155</u>, pp. 721-746

Vandervoort, Peter (1969)
Paper presented at IAU Symposium No. 38 (Basel) on "The Spiral Structure of Our Galaxy"

Table of Contents

I. Introduction

Quite often in the world of science, two apparently unrelated subjects of research may some day merge into a side-by-side interaction. Such an interaction is extremely important for the scientific development because it not only makes the independent check possible, but allows for new ideas which could never be reached individually. The purpose of the present set of tables is to provide a medium for the interaction of two of the most important developments in recent years in astronomy, Strömgren's photoelectric narrow-band photometry and the Lin-Shu density-wave theory of galactic spiral.

Like its predecessor, "Tables of Plane Galactic Orbits" by Contoupolos and Strömgren (1965), the present publication is prepared so that the place of origin for the moderately young stars now in the solar vicinity can be easily located. Since that publication, the spiral structure of the Milky Way System has become better understood both observationally and theoretically. A reliable spiral pattern (Lin, Shu, and Yuan, 1969; Yuan, 1969) has now been included in the present tables. The emphasis, therefore, is shifted to the understanding of the structure of the Milky Way System in a more quantitative fashion. The present set of tables is compiled for the following studies: (1) to establish definite evidence for the density wave theory, (2) to determine more precisely the pattern speed and the strength of the spiral gravitational field, (3) to initiate other studies concerning the local spiral structure, such as the vertex deviation and the Orion spur, and (4) to suggest the stellar ages, on the basis of the kinematic conditions, for those nearby stars to which Strömgren's method becomes relatively insensitive (e.g., those located in the lower half of the main sequence.)

The photoelectric narrow-band (Hβ and uvby) photometry was first introduced by Strömgren in the early fifties (Strömgren, 1951, 1952, 1956a, 1956b). Like many other systems of spectral classification, the purpose was to establish a method through which the essential characterisitics of a star can be obtained by observing a few selected indices derived from the properties of its optical spectrum. Because of its wide range of applicability and its simplicity in operation, this system has been widely adopted among the optical astronomers. One great advantage of this system, resulting from the recent developments by Strömgren and his collaborators (Crawford, Kelsall, Perry, et.al., see references) is its accuracy in the determination of the age and the photometric distance for the upper-half main-sequence stars in the solar vicinity. According to Strömgren (1967), the probable error in the age determination is narrowed to less than $\pm 15\%$ and the space velocity of a star calculated from its photometric distance only suffers an error less than 10%. With such a great accuracy, it is now meaningful to calculate the place of origin for the nearby stars.

Stars are generally believed to be formed in the spiral arm. The recent development of the density-wave theory has indeed demonstrated a process of star formation along the spiral arm, which agrees very well with all the observational evidence (Roberts 1969). Using the idea of this physical process, we can immediately set up an independent check between Strömgren's age determination and the density-wave theory. The history of migration of an individual star to its place of formation can be traced out with the kinematic properties specified by Strömgren's method. While the history of the individual star is being calculated, the spiral pattern at the same time is made to rotate backwards as a rigid body according to the density-wave theory. We can, therefore, check whether those nearby stars were indeed formed in the spiral arms of our Galaxy as expected. Or to be more precise, does there exist a pattern speed and a reasonable spiral gravitational field such that the nearby stars are found to be formed in the spiral arms of the theoretical pattern? The first study by Yuan (see Lin, Shu, Yuan 1969; Yuan 1969) based on the 25 late B stars whose kinematic conditions were determined by Strömgren showed that Strömgren's system and Lin's theory are completely compatible in this sense. The range of the pattern speed so obtained (12-14 km/sec-kpc), is consistent with the theoretical spiral pattern, and the range of strength of the spiral gravitational field (4-7% of the mean field) is also reasonable as checked by the systematic motion of neutral hydrogen in the galactic plane.

One important implication resulting from that study is that the spiral gravitational field, small as it is, plays a decisive role in deciding the birth place for the moderately young stars. It may cause a star to deviate from its true location of origin by 2 or 3 kpc along its own trajectory in the frame rotating at a given pattern speed. This is sufficient to bring the place of formation of a star from the spiral arm to the inter-arm region, and hence places a rather significant limit to the use of the tables of Contoupolos and Strömgren. To resolve this situation, the spiral gravitational field is included in the present calculations so that the original ideas of Contoupolos and Strömgren can be fully utilized. To include the spiral field is by no means a simple matter. It requires a rather complete knowledge of the spiral structure of the Milky Way, especially for the region 7 - 13 kpc from the galactic center. We shall not reproduce the detailed discussions here (see Yuan 1969). Some general feautres, however, will be summarized in Section II.

It might be noted that some of the results cited above are based on the behavior of the twenty-five stars. As more stars are analyzed, we shall expect more reliable and definite evidence in support of the density-wave theory. The present tables are, in particular, constructed for such purposes. Once the kinematic data of the nearby stars become available, we can, simply by interpolation with the aid of the tables, confirm the validity of the theory and find the appropriate values for the pattern speed and the spiral gravitational field of the Milky Way System without running into any elaborate computations.

The present tables also offer an opportunity to study the local structure of the Milky Way through an analysis of the statistics on the kinematics of the nearby young stars. Direct observations have so far produced a rather confused picture. The Orion spur, according to the optical observations, is inclined at a pitch angle of approximately 30°, which is difficult to fit into any large-scale spiral pattern of radio observations. Recent theoretical studies (Yuan 1969) reveal that a satisfactory picture in comparison with observations can be obtained if the optical Orion arm is treated as an inter-arm branch. (Such a picture has gained considerable support during the Basel Symposium 1969). However, more quantitative confirmation is still needed. It is conceivable that either the presence or the absence of the Orion arm as a major arm would cast appreciable difference in the behavior of the moderately young (mostly with low dispersion) stars now in the solar vicinity. This can be viewed from two different directions. First, consider the well known phenomenon of the vertex deviation. If the stars are uniformly formed in the galactic plane, the analysis of stellar dynamics would give an ellipsoidal distribution in the dispersion velocity, with its major axis pointing towards the anti-center direction for the nearby stars. Yet all the observations have shown that that axis is turned by about 25° from its original orientation. If the formation of the nearby stars were concentrated in the Perseus Arm and the Sagittarius Arm, we should expect some kind of deviation in the distribution of their dispersion velocity. Some preliminary investigations have indeed shown the plausibility of this approach (Lin, Strömgren and Yuan 1970). Secondly, we can estimate the influence of the Orion spur by analyzing the history of the migration of the relatively young stars, say, under 100 million years. These stars presumably have been affected by the gravitational field associated with the Orion spur for a longer portion of their lifetime. If their birth places, as traced back, are decidedly deviated away from the two major arms as determined by other studies cited above, we know that there is a substantial concentration of mass along the Orion spur. Otherwise, the Orion spur is probably just a luminous patch without great dynamical significance.

When the density-wave theory and Strömgren's method of age determination are sufficiently established with each other by the above procedure, the tables can be used to suggest the ages for those stars for which Strömgren's method becomes relatively inaccurate. The location and the space velocity at birth form a criterion for such suggestions. The favorable age for a star should be such that it was formed inside the spiral arm with a reasonable space velocity (less than 30 km/sec, say) in the neighborhood of the observed age.

The physical parameters and the theoretical model will be discussed in Section II. Some mathematical procedures are presented in Section III. The last section will be devoted to detailed description of the tables and diagrams as well as other functions.

II. Physical Parameters

(a) Spiral Pattern

Clearly, the decision on the adoption of the spiral pattern for the Milky Way System is the most difficult part in including the spiral graviational field in the calculations. There is no agreed spiral structure among the observers, nor any generally accepted theoretical spiral pattern. The density-wave theory, in fact, is the only theory which allows the construction of a large-scale spiral structure. Applying the scheme suggested by the theory, Lin and Yuan were able to obtain a two-arm spiral pattern which is also compatible with the observations (Lin, Shu and Yuan 1969; Yuan 1969a). It is based on the spiral pattern that the present calculations are carried out. The decision, as remarked, was not a simple one. A delicate balance between the theory and the observations must be maintained from time to time throughout the study. We shall refer the detailed discussions to the above cited papers.

The adopted spiral pattern is presented in Figure 1. One great advantage about this theoretical pattern is that it can be approximated almost perfectly by a logarithmic spiral pattern in the region of interest. The degree of perfection in the approximation is demonstrated in Figure 1. This simplifies our calculations tremendously. We only need to specify two parameters in order to define a theoretical pattern. They are the pitch angle and the reference point. The former determines the pattern and the latter fixes the orientation. To be more precise, consider the polar coordinates $(\tilde{\omega}, \theta)$, where $\tilde{\omega}$ is the galacto-centric distance measured in kpc and θ is the angle measured clockwise from the ray passing through the sun from the galactic center. A logarithmic two-arm spiral may be defined by the following equation

$$2\,(\theta-\theta_1) - \frac{2}{\tan i}\,\ln\,(\tilde{\omega}/\tilde{\omega}_1) = \text{constant} \qquad (1)$$

in which the pitch angle (>0) is denoted by $-i$ and the reference point $(\tilde{\omega}_1, \theta_1)$ is taken to be the center of the Sagittarius arm in the direction of the galactic center. For the present case, $i = -6^{\circ}.2$ and $(\tilde{\omega}_1, \theta_1) = (8.26,\ 0^{\circ})$.

(b) The Gravitational Field

Once the spiral pattern is specified, the gravitational potential is relatively easy to write out. The potential $V(\tilde{\omega}, \theta, t)$ has two components, the symmetrical part $V_o(\tilde{\omega})$ and the spiral part $V_1(\tilde{\omega}, \theta, t)$. We shall use the forcing function adopted by Contoupolos and Strömgren again for $V_o(\tilde{\omega})$, so that

$$\frac{\partial V_o}{\partial \tilde{\omega}} = \tilde{\omega}\Omega^2 = \frac{73340}{\tilde{\omega}^2} - 1581.8 + 3442.03\tilde{\omega} - 402.621\tilde{\omega}^2 + 12.9402\tilde{\omega}^3 \qquad (2)$$

in which Ω is the mean angular velocity of the galactic rotation. The rotation curve derived from the above equation agrees extremely well with the 1965 Schmidt model for the range 4-14 kpc. The spiral potential V_1, on the other hand, takes the following form in the context of the density-wave theory.

$$V_1 = A(\tilde{\omega}) \cos [2(\Omega_p t-\theta) + (\frac{2}{\tan i}) \ln (\frac{\tilde{\omega}}{\tilde{\omega}_1})] \qquad (3)$$

in which the sinusoidal variation is assumed. The amplitudes, $|A|$ is a slowly varying function of $\tilde{\omega}$ and will be treated as a constant here; Ω_p is the pattern speed; t is the time. The strength of the spiral gravitational field $|\partial V_1/\partial\tilde{\omega}| = 2A/\tan i$ is usually measured by the local mean field. Thus, in the following calculations, we shall refer to it as a quantity η defined as

$$\eta = 2A/(\tilde{\omega}_o \Omega_o{}^2 \tan i)$$

where the subscript denotes the value taken at the local standard of rest. As we have pointed out that the value of η is limited to a narrow range 4-7% from our previous studies, we shall choose four separate numbers equal to 4,5,6 and 7%. The theoretical spiral pattern (Fig. 1) is obtained at a pattern speed equal to 11.5 km/sec-kpc. However, a value 13.5 km/sec-kpc should be used in the actual case after the theory is modified by including the effects of the finite thickness and the presence of the gas (Shu 1968).

(c) The Pattern Speed

Strictly speaking, as the spiral pattern is chosen, we must use this pattern speed in the orbital calculations. Nevertheless, pattern speeds which are slightly different from the theoretical pattern speed will be allowed in order to account for the possible inaccuracy involved in the process of determining the theoretical pattern. As cited earlier, the range of the pattern speed is limited to 12-14 km/sec-kpc. Considering the volumes in the publication, we shall calculate only five cases 12, 12.5, 13, 13.5, and 14 km/sec-kpc.

(d) Present Kinematic Conditions

In order to carry out the numerical computations, we still have to specify the initial conditions of a star, including its present location and the space velocity, and finally a typical age we intend to cover. Due to the uncertainty in correcting the interstellar reddening, Strömgren's method becomes less accurate for the main-sequence stars much beyond 100 pc from us. This set of tables therefore would apply largely to those relatively nearby stars. For convenience, their present locations are all taken to be at the location of the sun, i.e., $(\tilde{\omega},\theta) = (10,0°)$. The errors caused by this approximation are insignificant as far as our present purposes are concerned.

It is well known that the space velocities of the early type stars are generally low. Their radial ($\tilde{\omega}$) and tangential (θ) components are usually less than 30 km/sec in absolute magnitude. Therefore, we shall take both present radial velocity and tangential velocity in the range from -30 to +30 km/sec. Contoupolos and Strömgren used as interval 10 km/sec in their tables. Since it makes the interpolation somewhat difficult, we have decided here to use an interval of 5 km/sec in order to improve the situation.

(e) The Range of the Stellar Age

Strömgren's method of age determination suffers an error of ±15%. In other words, a star with a specified age of 300 million years may be formed at time between 255 and 345 million years ago. A period of 90 million years normally would occupy a 3 kpc arc on its trajectory in the frame rotating at the given pattern speed, and it would be equally possible to place the star inside the spiral arm or in the inter-arm region. Any age beyond 300 million years simply makes this ambiguity even more pronounced. We shall, therefore, carry our calculations up to an age of 300 million years. It might be also noted that the limitation on the number of volumes to be published also contributes to this choice.

It should not be mistaken that the orbital calculations of this kind are useless for stars older than 300 million years. In fact, they may inform us of valuable statistics on the kinematics of nearby stars. Undoubtedly, such information would be more useful if other uncertainties such as the values of Ω_p and η, could be properly removed.

(f) Summary on the Adopted Physical Parameters

pattern speed Ω_p	12, 12.5, 13, 13.5, 14 km/sec-kpc
ratio of gravitational fields, η	4,5,6,7%
pitch angle $-i$	$6\overset{\circ}{.}2$
reference point $(\tilde{\omega}_1, \theta_1)$	(8.26, 0°)
forcing function	Contoupolos and Strömgren (1965)
present velocity, both radial and tangential	-30 - +30 km/sec, with an interval 5 km/sec
present location	(10,0°)
upper limit for the stellar age	300 million years

III. Orbital Calculations

Having decided on the spiral pattern and the initial (present) conditions of a star, we can immediately proceed to solve the equations of motion which, in the polar coordinates, are well known as follows

$$\frac{d^2\tilde{\omega}}{dt} = \tilde{\omega}\left(\frac{d\theta}{dt}\right)^2 - \frac{\partial V}{\partial \tilde{\omega}} \tag{4}$$

$$\frac{d}{dt}\left(\tilde{\omega}^2\frac{d\theta}{dt}\right) = -\frac{\partial V}{\partial \theta} \tag{5}$$

in which $V(\tilde{\omega},\theta,t) = V_o(\tilde{\omega}) + V_1(\tilde{\omega},\theta,t)$. From the concept of the density-wave theory, the spiral structure would be quasi-stationary looking from the frame rotating at the given pattern speed. It is therefore most convenient to treat the dynamical problem in that frame so that the potential is steady. In doing so, we shall introduce a new variable $\psi = \theta-\Omega_p t$. In terms of ψ, the equations of motion take the following form

$$\frac{d^2\tilde{\omega}}{dt} = \tilde{\omega}\left(\frac{d\psi}{dt} + \Omega_p\right)^2 - \left(\tilde{\omega}\Omega^2 - \tilde{\omega}_o\Omega^2_o\frac{\eta}{\tilde{\omega}}\sin\Delta\right) \tag{6}$$

$$\frac{d}{dt}\left[\tilde{\omega}^2\left(\frac{d\psi}{dt} + \Omega_p\right)\right] = \eta\tan i.\tilde{\omega}_o\Omega^2_o\sin\Delta \tag{7}$$

in which $\Delta = -2\psi - (2/\tan i)\ \ln(\tilde{\omega}/\tilde{\omega}_1)$.

The non-linear system is solved by the standard Runge-Kutta method with a step 10^6 years in t. The numerical results have been checked by two independent programmings; one at the computation center of the Massachusetts Institute of Technology and the other at the Institute for Space Studies, Goddard Space Flight Center, New York. Two additional checks were also performed. First by subdividing the integration time step and second by comparing results with tables of Contoupoles and Strömgren's when η was set to zero. Both cases assure all the figures in numerical results printed in the present set of tables.

In addition, diagrams of the spiral pattern and the trajectories with different present velocity are presented with the tables. The advantages of this presentation are that it not only gives us clear ideas on the history of migration of each individual star, in particular, their relative locations with respect to the spiral pattern, but also makes the interpolation job much simpler and more direct.

IV. Description of the Tables

As we have just mentioned, the publication is composed of three parts, (1) the introductory text, (2) the tables and (3) the diagrams. In this section we shall explain how to use the tables and the diagrams.

(a) Tables

This set of tables contains five volumes, corresponding to five different pattern speeds. Naturally, we have arranged them in the following way:

Vol. I	Ω_p = 12 km/sec-kpc
Vol. II	Ω_p = 12.5 km/sec-kpc
Vol. III	Ω_p = 13 km/sec-kpc
Vol. IV	Ω_p = 13.5 km/sec-kpc
Vol. V	Ω_p = 14 km/sec-kpc

In each volume, all four values of η are calculated. They appear on each page in the order upper left, upper right, lower left and lower right corresponding to η = 4,5,6 and 7% respectively. There are 169 pages of tables in each volume representing different combinations of present kinematic conditions. The results of the numerical calculations are printed out for every 10 million years.

The notation used in the tables is illustrated as follows:

T time in million years; the negative sign means in the past.

PI galactic-centric distance $\tilde{\omega}$, in kpc.

Theta angular displacement θ in degree in the fixed coordinate frame.

u space velocity along $\tilde{\omega}$, i.e., $d\tilde{\omega}/dt$ in km/sec.

v space velocity in the circumferential direction, i.e., $\tilde{\omega}\dot{\theta} - \tilde{\omega}\Omega$, in km/sec.

x x-axis of rectangular coordinates seated at the galactic center in the frame rotating at the given pattern speed. The axis is pointing east in the usual plane view of the Galaxy, and measured in kpc.

y y-axis of the above rectangular coordinates. It points towards the sun and is measured in kpc.

The alphabet after the age indicates the temporary location of the star. These letters designate the following different regions of the four major spiral arms.

	Phase Angle	Location
A	$-\frac{5}{2}\pi$ — -2π	Perseus Arm, Outer half
B	-2π — $-\frac{3}{2}\pi$	Perseus Arm, Inner half
C	$-\frac{\pi}{2}$ - 0	Sagittarius Arm, Outer half
D	0 - $\pi/2$	Sagittarius Arm, Inner half
E	$\frac{3}{2}\pi$ - 2π	Norma-Scutum Arm, Outer half

	Phase Angle	Location
F	$2\pi - \frac{5}{2}\pi$	Norma-Scutum Arm, Inner half
G	$\frac{7}{2}\pi - 4\pi$	Inner Arm (joining Sagittarius Arm), Outer half
H	$4\pi - \frac{9}{2}\pi$	Inner Arm (joining Sagittarius Arm), Inner half

As the spiral arm winds, it will merge into another arm after one full cycle. To avoid ambiguity, we have assigned C, for instance, to the region between two spiral lines specified by $\Delta = -\frac{\pi}{2}$ and $\Delta = 0$ respectively until this strip, winding counterwise, enters into the arm outside the Perseus at the location $\theta = 0°$ or $\psi = 0°$ again. Likewise, we have the correspondence between those letters and the phase angle Δ (See the above table).

(b) The diagrams

In each volume, 28 diagrams are presented to mark out the trajectories of the migration of stars with different initial conditions and different strengths of the spiral gravitational field. The results of Contoupolos and Strömgren strongly indicate that the trajectories of the migration of stars differ appreciatively from one another according to their present tangential space velocities, but not their radial space velocities. Therefore, for each diagram we fix the pattern speed, and the spiral field strength and the present radial velocity so that the nine trajectories traced out on the diagram represent respectively present tangential velocity equal to -30, -20, -10, 0, 10, 20, and 30 km/sec. The location of each star relative to the spiral pattern is indicated in each diagram, and a number is assigned to each trajectory to record the time. The interval for two successive numbers on the same trajectory corresponds to 50 million years.

The notation at the bottom of each diagram may be interpreted as follows:

ETA	η
ALPHA	$2/\tan i$
WO	$\Omega_{\circ}$
WP	Ω_p
PI_1	$\tilde{\omega}_1$

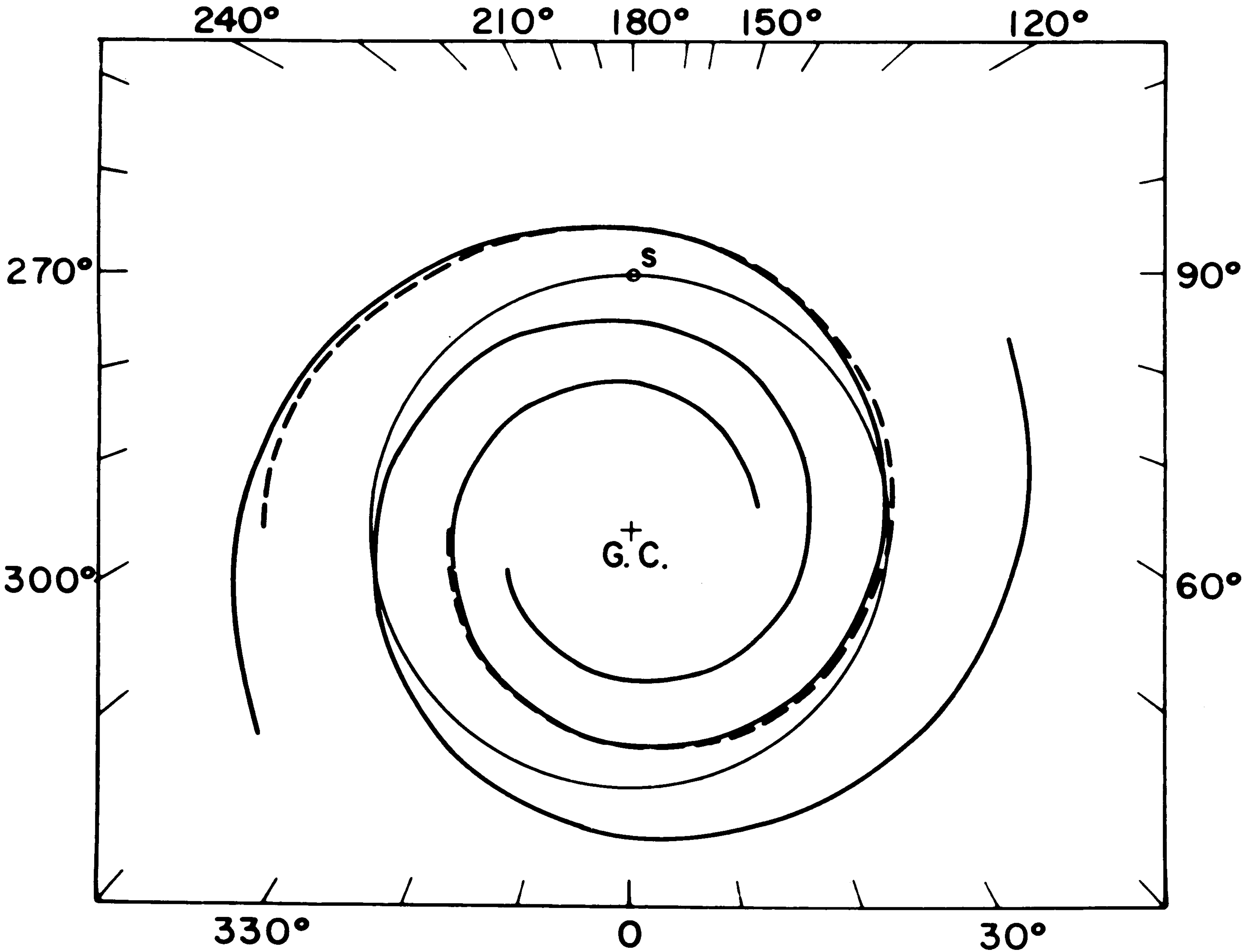
The shaded areas in the diagrams are the theoretical locations of the Perseus Arm, the Saqittarius Arm and the Norma-Scutum Arm and their extensions.

Acknowledgment

We would like to express our gratitude to Professor B. Strömgren and Professor C. C. Lin who initiated this program and have given constant encouragement and most valuable advice during the preparation of these tables. We also wish to thank Dr. R. Jastrow for his interest and support on this research program. The work at M.I.T. was supported by a grant from the National Aeronautics and Space Administration.

Caption of Figures

Figure 1. Comparison between the theoretical pattern constructed at the pattern speed equal to 11.5 km/sec-kpc (equivalent to 13.5 km/sec-kpc when the effects of finite thickness and the presence of the gas are included) and a logarithmic spiral pattern with a pitch angle equal to $6\overset{\circ}{.}2$ (dash line).


240°
210°
180°
150°
120°
270°
90°
S
G.C.
300°
60°
330°
0
30°

References

Contoupolos, G. and B. Strömgren, 1965, "Tables for Plane Galactic Orbits", Institute for Space Studies, NASA, N.Y.

Crawford, D. L., 1958, Ap. J., 128, 185.

Crawford, D. L. and B. Strömgren, 1966, Vistas in Astronomy, (London, Pergamon Press), 8., 149

Kelsall, T. and B. Strömgren, 1966, Vistas in Astronomy, (London, Pergamon Press), 8, 159.

Lin, C. C. and F. H. Shu, 1964, Ap. J., 140, 646.

____________________, 1966, Proc. Nat. Acad. Sci., 55, 229.

Lin, C. C., F. H. Shu, and C. Yuan, 1969, Ap. J., 155, 721.

Lin, C. C., B. Strömgren and C. Yuan (in preparation).

Roberts, W. W., 1969, Ap. J., 158, 123.

Shu, F. H., unpublished Ph. D. Thesis (Harvard 1968)

Schmidt, M., 1965, in Galactic Structure, ed. A. Blaauw and M. Schmidt, (University of Chicago Press), p. 513.

Strömgren, B., 1951, A. J., 56, 142.

______________, 1952, A. J., 57, 200.

______________, 1956a, in Proceedings of the Third Berkeley Symposium, ed. J. Neynan, (Berkeley, Calif., University of California Press), 3, 49.

______________, 1956b, in Vistas in Astronomy, (London, Pergamon Press), 2, 1336.

__________________, 1963a, in Basic Astronomical Data, ed. K. A. Strand, (Chicago, University of Chicago Press), p. 123.

__________________, 1963b, Quart. J. Roy. Astron. Soc., 4, 8.

__________________, 1964, Astrophysica Norvegica, 9, 333.

__________________, 1966a, Annual Review of Astronomy and Astrophysics, 4, 433.

__________________, 1966b, in Stellar Evolution, ed. R. F. Stein and A. G. W. Cameron (New York, Plenum Press), p. 391.

__________________, 1967, in Proc. I.A.U. Symp. No. 31, p. 323.

Yuan, C., 1969a, Ap. J., 158, 871.

__________________, 1969b, Ap. J., 158, 889.